Nonlinear Systems

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August 30, 2022

Chapter 2: Second Order Systems

Concept of Phase Plane

A second-order autonomous system is represented by two scalar differential equations

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2) \end{aligned} \tag{1}$$

Let $x(t) = (x_1(t), x_2(t))$ be the solution of (1) that starts at a certain initial state $x_0 = (x_{10}, x_{20})$.

f(.) is called a **vector field**

The set of points $\{(t, x_1(t), x_2(t)); t \in \mathbb{R}\}$ with (x_1, x_2) a solution of (1) (and $x_1(t_0) = x_{10}$ and $x_2(t_0) = x_{20}$ for some t_0) is called the **trajectory** or **solution curve** (through (x_{10}, x_{20})).

The set of points $\{(x_1(t), x_2(t)); t \in \mathbb{R}\}$ with (x_1, x_2) a solution of (1) (and $x_1(t_0) = x_{10}$ and $x_2(t_0) = x_{20}$ for some t_0) is called the $fOrbit$ or **Phase Curve** (through (x_{10}, x_{20})).

Concept of Phase Plane

An orbit that forms a closed curve is called a **closed orbit**.

The family of all trajectories of a dynamical system is called **the phase portrait.**