Chapter V : Poisson's Equation in Semiconductor Junctions

V.1. Introduction

When n-type and p-type semiconductors are joined, mobile charge carriers (electrons and holes) diffuse across the boundary and recombine. This process creates a zone with no free carriers, called the **depletion region** or **space charge region**.

The way the doping concentration changes across this junction can be described using different models. The two most common are the **abrupt junction** and the **linearly graded junction**. The abrupt junction is an idealized model where the doping concentration changes instantaneously from p-type to n-type. This simplifies the math needed to analyze the junction's electrical properties, such as its electric field, potential, and capacitance. Because of its simplicity, the abrupt junction model is a fundamental concept in semiconductor physics and is essential for understanding the behavior of electronic devices such as diodes, transistors and solar cells.

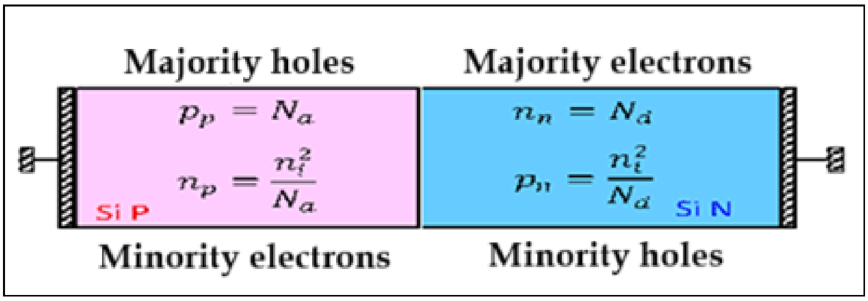

Figure V.1. Contact between p-type and n-type semiconductor

V.2 Electrostatic modeling in PN junctions

To understand the internal electric field and potential profile within a semiconductor device, particularly across a PN junction, Poisson's equation is essential. It connects the distribution of electric charge to the resulting electrostatic potential and field, making it a cornerstone for analyzing depletion regions and junction behavior under equilibrium and bias.

V.2. 1 Formation of the space charge region (SCR)

When a p-type and n-type semiconductor are joined:

- Electrons from the n-region diffuse into the p-region and recombine with holes.
- Holes from the p-region diffuse into the n-region and recombine with electrons.
- This leads to the creation of fixed charges on both sides of the junction, forming the SCR or depletion region.

This redistribution results in:

- Positive space charge on the n-side
- Negative space charge on the p-side

generating an internal electric field and an electrostatic potential.

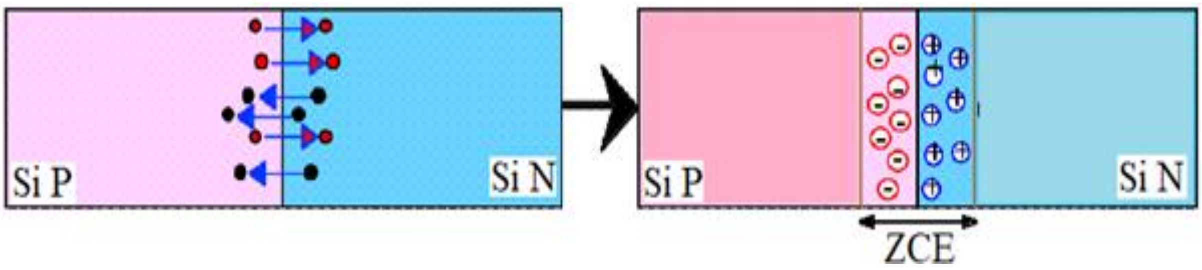

Figure V. 2. Diffusion/recombination and formation of the depletion region

V.2. 2 Poisson's equation and electric field relations

- **Mathematical formulation**

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_0 \varepsilon_r}$$

The electric field is related to the potential by:

$$\xi(x) = -\nabla V(x) = -\frac{dV(x)}{dx}$$

Where:

- $\rho(x) = q(p - n + N_D - N_A)\ (\frac{Cb}{cm^3})$
- $\varepsilon_r = 11{,}8$: relative dielectric permittivity of Silicon
- $\varepsilon_0 = 8{,}854 \times 10^{-14} F/cm$: absolute permittivity of vacuum.

- **Charge density profile**

Assuming complete ionization:

1. **ρ(x) = 0**, for : **$x \leq -x_p$** and **$x \geq x_n$**
2. **ρ(x) = - qN_A**, for : **$-x_p < x \leq 0$**
3. **ρ(x) =qN_D**, for: **$0 \leq x < x_n$**

- **Electric field expressions**

Integrating Poisson's equation yields:

$\xi(x) = \frac{1}{\varepsilon_0 \varepsilon_r} \int_{-\infty}^{+\infty} \rho(x).dx$

1. $x \leq -x_p$ et $x \geq x_n$: $\xi(x) = 0$
2. **P-side:** $-x_p < x \leq 0$:

$$\xi(x) = \frac{1}{\varepsilon_0 \varepsilon_r} \int -\mathrm{qN}_A.dx = \frac{-\mathrm{qN}_A}{\varepsilon_0 \varepsilon_r} x + C$$

For : $x = -x_p$: $\xi(x) = 0$;

$$\xi(x) = \frac{\mathrm{qN}_A}{\varepsilon_0 \varepsilon_r} x_p + C = 0$$

From which : $C = -\frac{\mathrm{qN}_A}{\varepsilon_0 \varepsilon_r} x_p$

Therefore:

$$\xi(x) = \frac{-\mathrm{qN}_A}{\varepsilon_0 \varepsilon_r}(x_p + x)$$

3. **N-side:** $0 \leq x < x_n$:

$\xi(x) = \frac{1}{\varepsilon_0 \varepsilon_r} \int \mathrm{qN}_D.dx = \frac{\mathrm{qN}_D}{\varepsilon_0 \varepsilon_r} x + C$

For : $x = x_n$: $E(x) = 0$;

$$\xi(x) = \frac{qN_D}{\varepsilon_0 \varepsilon_r} x_n + C = 0$$

D'où : $C = -\frac{qN_D}{\varepsilon_0 \varepsilon_r} x_n$

Therefore:

$$\xi(x) = \frac{qN_D}{\varepsilon_0 \varepsilon_r}(x - x_n)$$

Where: $-x_p$ and x_n are the boundaries of the depletion region.

The electric field must be continuous at the point x = 0, consequently:

$\frac{-qN_A}{\varepsilon_0 \varepsilon_r} x_p = -\frac{qN_D}{\varepsilon_0 \varepsilon_r} x_n \Longrightarrow N_A x_p = N_D x_n$

- **Potential distribution**

Integrating the electric field:

$$V(x) = -\int_{-\infty}^{+\infty} \xi(x) \, . dx$$

1- **P-side:** pour - $x_p < x \leq 0$:

$$V(x) = \frac{qN_A}{\varepsilon_0 \varepsilon_r} \int_{-x_p}^{x} (x_p + x)\, dx = \frac{qN_A}{2\varepsilon_0 \varepsilon_r}(x_p + x)^2$$

2- **N-side:** , pour $0 \leq x < x_n$, given that $\boldsymbol{V(x_n) = V_{bi}}$, we have :

$V(x) = \frac{qN_D}{\varepsilon_0 \varepsilon_r} \int_x^{x_n} (x_n - x). dx = -\frac{qN_D}{2\varepsilon_0 \varepsilon_r}(x_n - x)^2 + V_{bi}$

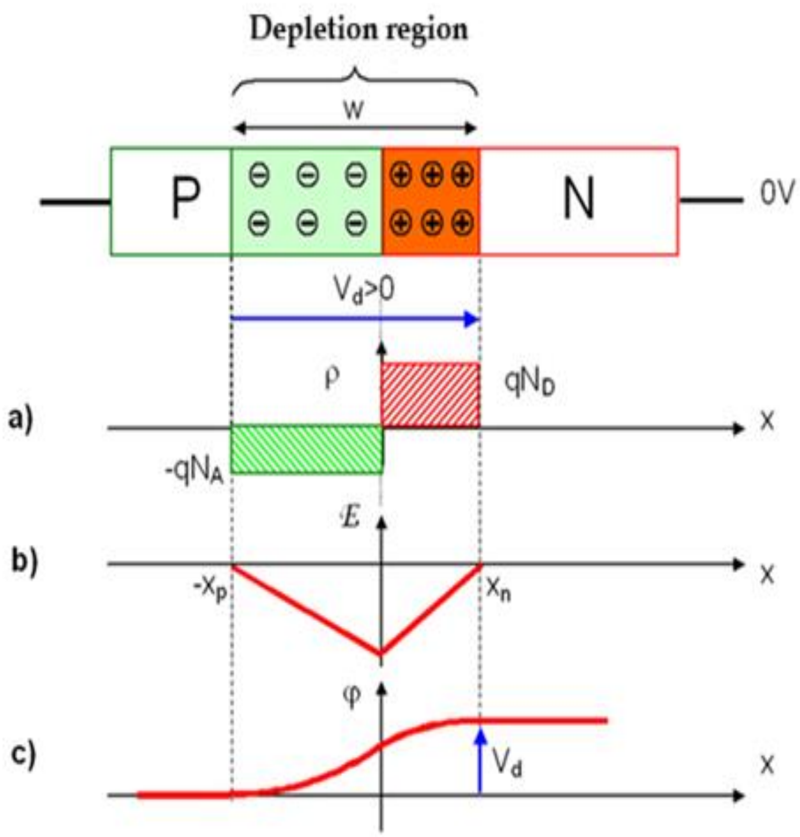

Figure V. 3. Spatial profiles of PN junction at equilibrium: a)Charge density, b)Electric field, c)Potential.

V.2. 3 Depletion region width and related quantities

- Depletion width on p-side: $x_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}.V_{bi}\frac{N_D}{N_A(N_A+N_D)}}$
- Depletion width on n-side : $x_n = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}.V_{bi}\frac{N_A}{N_D(N_A+N_D)}}$

Note that when N_A increases (with N_D constant), x_p decreases. And when N_D increases (with N_A constant), x_n decreases.

- Total depletion width:

$$W = x_n - (-x_p) = x_n + x_p$$

$$W = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}.V_{bi}\frac{N_A + N_D}{N_A.N_D}}$$

- **Maximum electric field**

$$\xi_{max} = |\xi(x = 0)| = \frac{qN_A}{\varepsilon_0 \varepsilon_r} x_p = \frac{qN_D}{\varepsilon_0 \varepsilon_r} x_n = \frac{2V_{bi}}{W}$$

Example :

Parameters :

- $n_i = 10^{10}$ /cm^3 ,
- $N_A = 10^{16}$/cm^3,
- $N_D = 10^{15}$/cm^3 ;
- KT/q = 0,026 eV à 300 K.

Results :

- x_n = 0,8845 µm ;
- x_p = 0 ,08845 µm ;
- W = 0,973 µm.

V.2. 4 Junction capacitance (transition capacitance)

As previously mentioned, a depletion region forms at the junction, bounded by positive charges on the N-side and negative charges on the P-side; consequently, it behaves like a parallel-plate capacitor:

$$C = \frac{\varepsilon_0 \varepsilon_r S}{W}$$

With :

$$W = \sqrt{\frac{2\varepsilon_0 \varepsilon_r (N_A + N_D)}{qN_A N_D} V_{bi}}$$

Substitude W :

$$C = \varepsilon_0 \varepsilon_r S \sqrt{\frac{qN_A N_D}{2\varepsilon_0 \varepsilon_r (N_A + N_D) V_{bi}}} = S \sqrt{\frac{q\varepsilon_0 \varepsilon_r}{2} . \frac{N_A N_D}{(N_A + N_D) V_{bi}}}$$

Where :

$$V_{bi} = \frac{KT}{q} ln\left(\frac{N_A N_D}{n_i^2}\right) = U_T ln\left(\frac{N_A N_D}{n_i^2}\right)$$

V.2. 5 Effects of biasing on Poisson's equation

When an external DC voltage is applied across a PN junction, it alters the energy landscape of the junction by modifying the height of the potential barrier (built-in potential):

- Under **forward bias**, the external voltage is applied in such a way that it opposes the built-in potential. As a result, the effective barrier height decreases. This reduction facilitates the movement of majority carriers across the junction, increasing the diffusion current. The depletion region narrows, and the energy bands flatten near the junction, reflecting reduced opposition to carrier flow.
- Under **reverse bias**, the external voltage adds to the built-in potential, increasing the barrier height. This suppresses the diffusion of majority carriers and widens the depletion region. Consequently, the flow of current is greatly reduced, limited to a small leakage current due to minority carriers.

This modulation of the barrier height is crucial for the operation of diodes and explains the rectifying behavior of the PN junction. It also influences capacitance, carrier injection, and breakdown characteristics in more complex devices like Zener diodes and bipolar transistors.

In summary:

- **Forward bias** reduces the built-in potential $\boldsymbol{V_{bi}}$, shrinking **W** and increasing capacitance.
- **Reverse bias** increases $\boldsymbol{V_{bi}}$, expanding **W** and reducing capacitance.
- Under bias, replace $\boldsymbol{V_{bi}}$ with $\boldsymbol{V_{bi} - V}$ in all related equations.

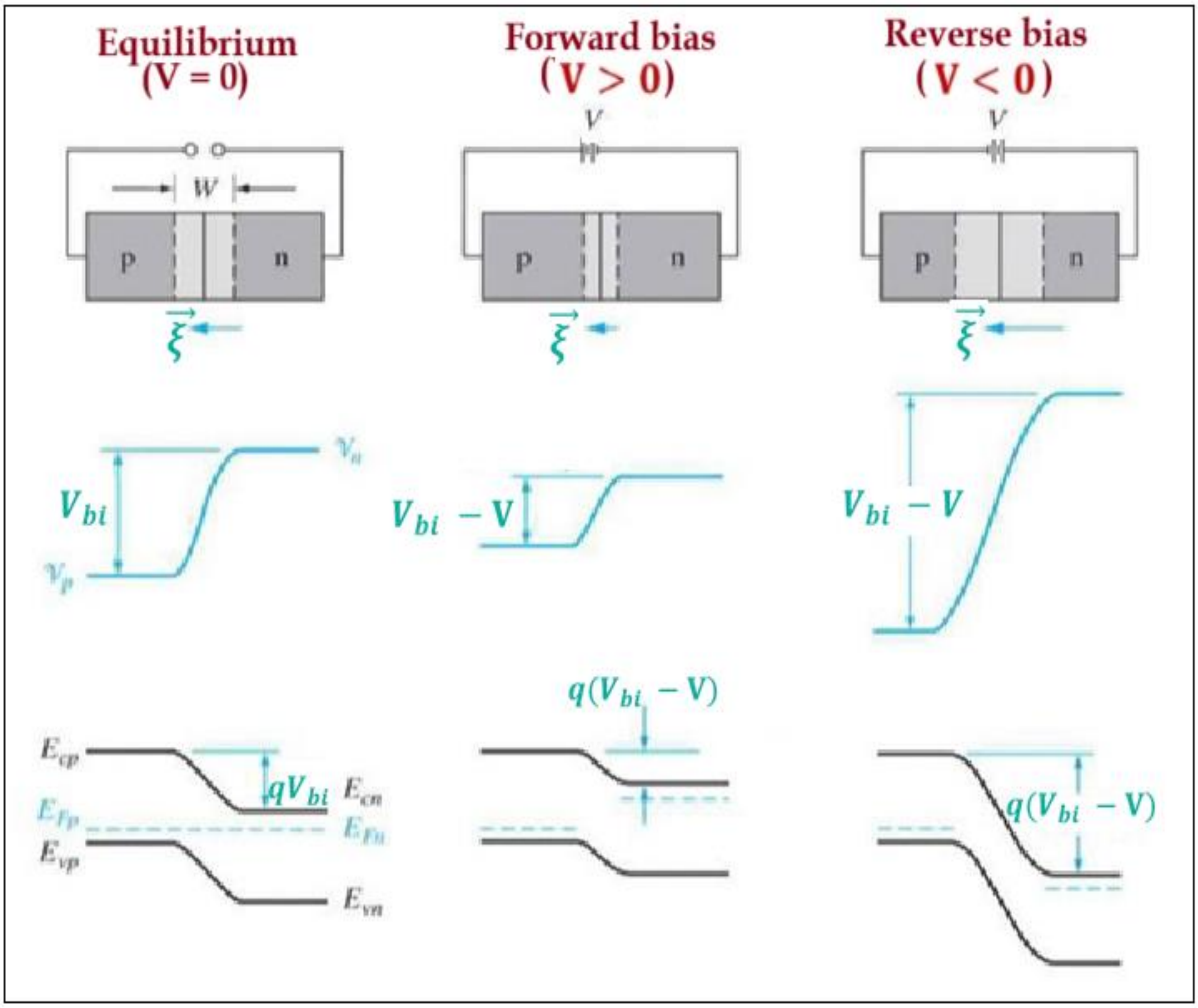

Figure V. 4. Equilibrium, Forward Bias, and Reverse Bias in a P-N Junction