# *Chapter VI : Low Non-Equilibrium Perturbations (Charge Transport)*

## VI.1. Introduction – Overview of charge transport mechanisms

In an intrinsic semiconductor where $\mathbf{n = p = n_i}$, and in the absence of external excitation, the system remains in thermodynamic equilibrium. At absolute zero **(0 K)**, all electrons occupy the valence band **(VB)**, rendering the semiconductor a perfect insulator. However, at higher temperatures, thermal energy is transferred to the crystal, allowing some electrons to gain enough energy to transition from the valence band to the conduction band **(CB)**, provided the absorbed energy exceeds the bandgap energy $\mathbf{E_g}$.

Once in the conduction band, electrons possess an average thermal energy that corresponds to a thermal velocity $\mathbf{V_{th}}$. While not all electrons move at the same velocity due to the Pauli exclusion principle, the kinetic theory of gases enables estimation of their average kinetic energy:

$$E_{kin} = \frac{1}{2} m^* V_{th}^2 = \frac{3}{2} KT$$

At 300 K, this yields a thermal velocity of approximately $10^5$ $ms^{-1}$. The average distance an electron travels between two collisions is called the mean free path λ:

$$\lambda = V_{th} \tau$$

where τ is the mean free time between collisions. At room temperature, this path length is around 100A° = 10 nm.

## VI.2. Carrier mobility and drift velocity

When an electric field $\xi(V/m)$ is applied to a semiconductor, charge carriers experience two forces:

- An electrostatic force: $\vec{F} = \pm q\vec{\xi}$ (+ for electrons and – for holes);
- A frictional force from collisions: $-f\overrightarrow{V_d}$

This results in the following acceleration: (Force = mass × acceleration):

$$\vec{\gamma} = \frac{d\vec{V}_d}{dt} = \pm\frac{q}{m^*}\vec{\xi} - \frac{f}{m^*}\vec{V}_d = \pm\frac{q}{m^*}\vec{\xi} - \frac{\vec{V}_d}{\tau}$$

Where:

- $m^*$ is the mass of the carrier;
- $\vec{V}_d$ is its drift velocity (or simply drift);
- τ represents viscous-like friction.

Under steady-state conditions (i.e., when acceleration is zero: $\vec{\gamma} = 0$):

$$\vec{V}_d = \pm\frac{q\tau}{m^*}\vec{\xi} = \pm\mu\vec{\xi}$$

Thus:

- Electron drift velocity:

$$\vec{V}_{dn} = -\frac{q\tau}{m_n}\vec{\xi} = -\mu_n\vec{\xi}$$

- Hole drift velocity:

$$\vec{V}_{dp} = \frac{q\tau}{m_p}\vec{\xi} = \mu_p\vec{\xi}$$

Carrier mobilities are defined as:

$$\mu_n = \frac{q\tau}{m_n}, \qquad \mu_p = \frac{q\tau}{m_p} \qquad (\,cm^2V^{-1}s^{-1})$$

The following table gives the **mobility values of electrons and holes** for several **semiconductors**.

*Table VI.1 Mobility of electrons, holes, band gap energy, and permittivity for some semiconductors*

| SC | $E_G$ | $\lvert\mu_n\rvert$ | $\mu_p$ | $\varepsilon_r$ | SC | $E_G$ | $\lvert\mu_n\rvert$ | $\mu_p$ | $\varepsilon_r$ |
|---|---|---|---|---|---|---|---|---|---|
| Si | 1.12 | 1400 | 500 | 11.9 | InAs | 0.36 | 33000 | 460 | 14.6 |
| Ge | 0.7 | 3900 | 1900 | 16 | InSb | 0.18 | 78000 | 750 | 17.7 |
| GaAs | 1.42 | 8500 | 400 | 13 | GaSb | 0.72 | 5000 | 850 | 15.7 |
| InP | 1.35 | 5000 | 150 | 12.4 | CdS | 2.42 | 340 | 50 | 5.4 |
| AlAs | 2.16 | 1200 | 400 | 10.1 | CdTe | 1.56 | 1050 | 100 | 10.2 |
| GaP | 2.26 | 300 | 100 | 11 | $SiO_2$ | 9 | isolant | | 3.9 |

## VI.3. Electrical conductivity

The conduction current densities **(A/m²)** are:

- For electrons: $\vec{J}_n^c = -qn\vec{V}_n = qn\mu_n\vec{\xi} = \sigma_n\vec{\xi}$

With : $\vec{V}_n = -\mu_n\vec{\xi};$

- For holes: $\vec{J}_p^c = qp\vec{V}_p = qp\mu_p\vec{\xi} = \sigma_p\vec{\xi}$

With: $\vec{V}_p = \mu_p\vec{\xi}$

$\sigma_n = qn\mu_n(\Omega\mathbf{m})^{-1}$ and $\sigma_p = qn\mu_p(\Omega\mathbf{m})^{-1}$ **or (S/m)**

Total conduction current densities is:

$$\vec{J}_T^c = \vec{J}_n^c + \vec{J}_p^c = (\sigma_n + \sigma_p)\vec{\xi} = \sigma_T\vec{\xi}$$

Resistivity: $\rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$

➢ Special cases:

- **Intrinsic semiconductors**: $n_0 = p_0 = n_i$ : $\vec{J}_T^c = qn_i(\mu_n + \mu_p)\vec{\xi}$

- Intrinsic resistivity : $\rho_i = \frac{1}{\sigma_n} = \frac{1}{qn_i(\mu_n+\mu_p)}$

With :

$$n_i = \sqrt{N_c N_v}e^{\frac{-E_g}{2KT}}$$

**Notes :**

- For **N-type semiconductors**: ( $n = N_D$), $p \ll n$ : $\sigma = \sigma_n + \sigma_p \approx qN_D\mu_n$, and $\vec{J}_T^c = \vec{J}_n^c$
- For **P-type semiconductors**: ( $p = N_A$), $n \ll p$ : $\sigma = \sigma_n + \sigma_p \approx qN_A\mu_p$, and $\vec{J}_T^c = \vec{J}_p^c$.

### VI.3. Carrier diffusion

A semiconductor contains two types of mobile charge carriers: **electrons and holes**. These carriers can move due to two main effects:

- **An electric field**, producing a **drift (conduction) current.**
- **A concentration gradient**, leading to a **diffusion current**.

Diffusion occurs whenever there is a concentration gradient of carriers in the crystal. These gradient causes carriers to move from high-concentration regions to low-concentration regions, a net movement that works to make the concentration uniform throughout the material.

The carrier fluxes, $F_n$ and $F_p$ ($m^{-3}s^{-1}$), are given by **Fick's law**:

- **For an N-type semiconductor:**

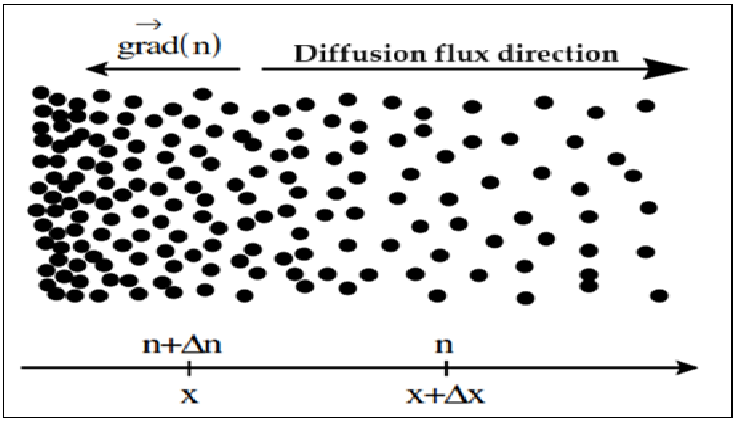
$$\vec{F}_n = -D_n\frac{dn}{dx} = -D_n grad(n)$$

- **For a P-type semiconductor:**

$$\vec{F}_p = -D_p \frac{dn}{dx} = -D_p grad(p)$$

Where $D_n$ and $D_p$ ($m^2s^{-1}$) are the diffusion coefficients for electrons and holes, respectively.

The negative sign indicates that the flux direction is opposite to the concentration gradient.




***Figures VI.1. The electron concentration gradient causes these electrons to diffuse toward the region with lower concentration.***

The diffusion current densities are:

- For electrons: $\vec{J}_n^D = -q\vec{F}_n = qD_n grad(n)$
- For holes: $\vec{J}_p^D = q\vec{F}_p = -qD_p grad(p)$

➢ **Carrier conduction and diffusion:**

For a semiconductor with a non-uniform concentration and to which an electric field, E, is applied, we can write the electron current density by accounting for both diffusion and electrical conduction.

- The electron current density is therefore:

$$\vec{J}_n = \vec{J}_n^D + \vec{J}_n^c = qn\mu_n\vec{\xi} + qD_n \frac{dn}{dx}$$

- Similarly, the hole current density is:

$$\vec{J}_p = \vec{J}_p^D + \vec{J}_p^c = qp\mu_p\vec{\xi} - qD_p\frac{dp}{dx}$$

Consequently, the total current density is given by:

$$\vec{J}_T = \vec{J}_n + \vec{J}_p$$

The diffusion coefficients are related to mobility by Einstein's relation:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{KT}{q}$$

The term : $U_T = \frac{KT}{q}$ is the thermal voltage, typically 26mV at 300K.

## VI.4. Displacement current in time-varying fields:

In time-varying (AC) conditions, a displacement current arises and is given by:

$$\vec{J}_d = \frac{\partial\vec{D}}{\partial t}$$

Where $\vec{D}$ is the electric displacement vector:

$$\vec{D} = \varepsilon\vec{\xi} = \varepsilon_0\varepsilon_r\vec{\xi}$$

If the electric field varies sinusoidally, $\vec{\xi} = \vec{\xi}_0 e^{jwt}$, then :

$$\vec{J}_d = jw\varepsilon\vec{\xi}$$

**Note:** The displacement current will only have a significant amplitude at frequencies on the order of GHz. It is generally considered negligible, except at high frequencies (HF) during the analysis of time-varying conditions.

## VI.5. Charge transport in electric and magnetic fields – Hall effect and magnetoresistance:

When a semiconductor is subjected to both an electric field and a magnetic field, the motion of charge carriers is influenced by the **Lorentz force**, which alters the direction of current flow. This gives rise to two significant phenomena: The **Hall effect** and **magnetoresistance**.

In the Hall effect, when a magnetic field $\vec{\boldsymbol{B}}$ is applied perpendicular to the direction of current $\vec{\boldsymbol{I}}$ in a semiconductor, it exerts a force on the moving charge carriers (electrons or holes), causing them to accumulate on one side of the material. This separation of charges generates a transverse electric field called the **Hall field**, $\vec{\boldsymbol{E}}_{\boldsymbol{H}}$, which creates a measurable voltage across the width of the material—known as the **Hall voltage** $\boldsymbol{V_H}$. The sign and magnitude of $\boldsymbol{V_H}$ depend on the type and concentration of charge carriers and the strength of the magnetic field. The Hall voltage is expressed as:

$$\boldsymbol{V_H} = \frac{IB}{qnd}$$

Where:

- **I** is the current;
- **B** is the magnetic field strength;
- **q** is the charge of the carrier;
- **n** is the carrier concentration;
- **d** is the thickness of the semiconductor.

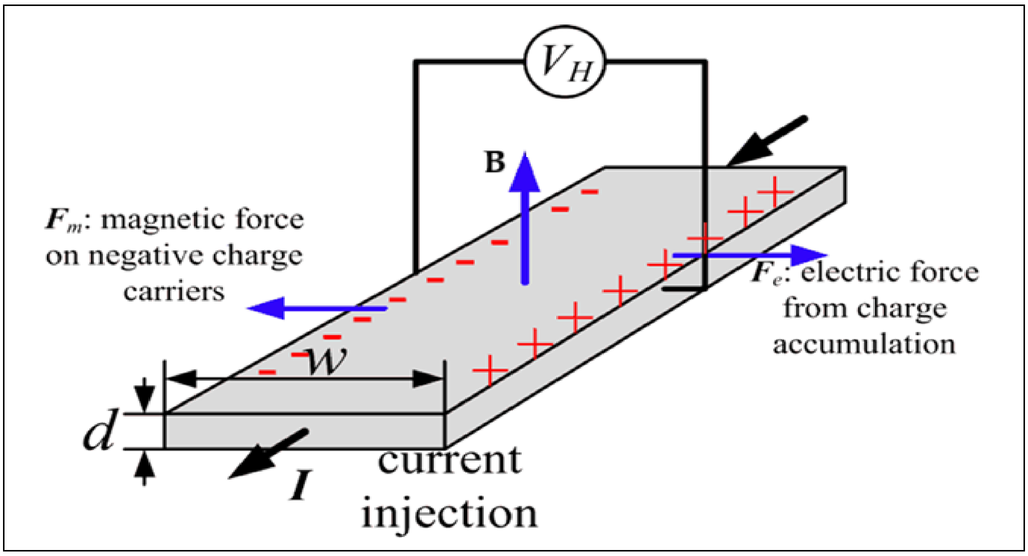
This effect allows for the determination of the type of semiconductor (n-type or p-type), carrier concentration, and mobility, making it a valuable diagnostic tool in semiconductor physics.

**Magnetoresistance**, on the other hand, refers to the change in electrical resistance of a material when exposed to a magnetic field. The Lorentz force causes charge carriers to follow curved paths, increasing the likelihood of collisions and effectively increasing the resistance. This resistance change depends on the magnetic field strength and the mobility of the carriers. For low magnetic fields, the change in resistance $\Delta \boldsymbol{R}$ is approximately proportional to the square of the magnetic field:

$$\frac{\Delta R}{R_0} \propto (\mu B)^2$$

Where $\boldsymbol{\mu}$ is the mobility and **B** is the magnetic flux density. While magnetoresistance is generally weak in non-magnetic semiconductors, it becomes significant in materials with high mobility or in devices designed to exploit this effect, such as magnetic sensors and memory elements.

In conclusion, the combined action of electric and magnetic fields on charge transport reveals essential characteristics of semiconductor materials and enables the development of Hall sensors and magnetoresistive devices widely used in electronics and sensing applications.




*Figures VI.2. Schematic representation of the Hall effect*