Chapter VIII: Space – Time Evolution Equations

VII.1 Introduction

This chapter presents the fundamental equations that govern the spatial and temporal evolution of carrier concentrations in semiconductors. These equations are essential for analyzing transport phenomena, device response times, and the behavior of materials under non-equilibrium conditions.

VII.2 Continuity equations (or conservation equations for each carrier type)

The continuity equation describes how the concentration of charge carriers (electrons or holes) changes with time and position. It is derived from the principle of conservation of particles and accounts for current flow.

1- Assuming only transport:

- **For electrons:**

$$\frac{\partial n}{\partial t} = \frac{1}{q}.div\overrightarrow{J_n} = div\left(n\mu_n\vec{\xi}\right) + div(D_n grad(n))$$

- **For holes:**

$$\frac{\partial p}{\partial t} = \frac{1}{q}.div\overrightarrow{J_p} = -\,div\left(p\mu_p\vec{\xi}\right) + div(D_p grad(p))$$

With current density $\overrightarrow{J_n}$ and $\overrightarrow{J_p}$ (drift + diffusion):

$$\overrightarrow{J_n} = qn\mu_n\vec{\xi} + qD_n grad(n)$$

$$\overrightarrow{J_p} = qp\mu_p\vec{\xi} + qD_p grad(p)$$

2- If generation and recombination phenomena are also taken into account within the semiconductor, the continuity equations become:

- **For electrons:**

$$\frac{\partial n}{\partial t} = div\left(n\mu_n\vec{\xi}\right) + div(D_n grad(n)) + G_n - R_n$$

- **For holes:**

$$\frac{\partial p}{\partial t} = -\, div\left(p\mu_p\vec{\xi}\right) + div(D_p grad(p)) + G_p - R_p$$

These two **continuity equations** for electrons and holes take into account four main contributions:

1. Variation due to carrier drift under the action of an electric field $\vec{\xi}$;
2. Variation due to carrier diffusion caused by spatial concentration gradients;
3. Increase in carrier concentration due to generation processes **(G_n, G_p)**;
4. Decrease in carrier concentration due to recombination processes **(R_n, R_p)**.

3- For a One-Dimensional model:

- **For electrons:**

The continuity equation becomes:

$$\frac{dn}{dt} = \mu_n \frac{\partial n\xi}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + G_n - \frac{n - n_0}{\tau_n} = D_n \frac{\partial^2 n}{\partial x^2} + n\mu_n \frac{\partial \xi}{\partial x} + \xi\mu_n \frac{\partial n}{\partial x} + G_n - \frac{n - n_0}{\tau_n}$$

- **For holes:**

$$\frac{dp}{dt} = -\mu_p \frac{\partial p\xi}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + G_p - \frac{p - p_0}{\tau_p}$$

$$= D_p \frac{\partial^2 p}{\partial x^2} - p\mu_p \frac{\partial \xi}{\partial x} - \xi\mu_p \frac{\partial p}{\partial x} + G_p - \frac{p - p_0}{\tau_p}$$

Physical meaning of terms :

- $D_n \frac{\partial^2 n}{\partial x^2}$ ($D_p \frac{\partial^2 p}{\partial x^2}$): diffusion of electrons (holes);
- $n\mu_n \frac{\partial \xi}{\partial x}$ ($p\mu_p \frac{\partial \xi}{\partial x}$): drift due to the **spatial variation of the electric field;**
- $\xi\mu_n \frac{\partial n}{\partial x}$ ($\xi\mu_p \frac{\partial p}{\partial x}$): drift due to the **carrier gradient in the presence of an electric field;**

- $G_n(G_p)$: generation rate of electrons (holes);
- $\frac{n-n_0}{\tau_n}(\frac{p-p_0}{\tau_p})$: recombination rate (excess carriers over equilibrium).

VII.3 Charge conservation equation

The total charge in a semiconductor system must be conserved. By combining the electron and hole continuity equations, we obtain the **charge continuity equation**:

$$\frac{\partial\rho}{\partial t} + div\vec{J} = 0$$

Where:

- $\boldsymbol{\rho}$ is the charge density:

$$\rho = q(p - n + N_D^+ - N_A^-)$$

- $\vec{\boldsymbol{J}} = \vec{\boldsymbol{J}}_{\boldsymbol{n}} + \vec{\boldsymbol{J}}_{\boldsymbol{p}}$: total current density

This equation ensures that the net current flow and changes in local charge density are consistent with the laws of electrostatics and Maxwell's equations.

VII.4 "Ambipolar" (or Generalized) continuity equation

In many practical situations, especially in low-level injection and when electrons and holes move together (e.g., in intrinsic or lightly doped semiconductors), it is useful to combine the two continuity equations into a **single ambipolar equation**.

This equation describes the **coupled transport** of electrons and holes under the assumption that their generation and recombination rates are the same ***(i.e., Δn = Δp)***:

$$\frac{\partial\Delta n}{\partial t} = D_a grad^2(\Delta n) - \frac{\Delta n}{\tau}$$

Where:

- $\boldsymbol{\Delta n}$: excess carrier concentration
- $\boldsymbol{D_a}$: ambipolar diffusion coefficient

$$D_a = \frac{nD_p + pD_n}{n + p}$$

- $\boldsymbol{\tau}$ **:** effective carrier lifetime

This equation is particularly useful for modeling optical excitation and diffusion of minority carriers.

VII.5 Application examples

VII.5.1. Life time and diffusion length

The **carrier lifetime τ** represents the average time an excess carrier exists before recombining. During this time, a carrier can diffuse a certain distance, known as the **diffusion length L**:

$$L_n = \sqrt{D_n \tau_n}, \qquad L_p = \sqrt{D_p \tau_p},$$

Where D_n and D_p are the diffusion coefficients for electrons and holes. These parameters are key in determining how far carriers travel before they recombine, influencing the performance of devices such as photodiodes and solar cells.

VII.5.2. Dielectric relaxation time and Debye length

Dielectric relaxation time τ_ρ represents the time it takes for a semiconductor to neutralize a local charge imbalance through redistribution of carriers:

$$\tau_\rho = \frac{\varepsilon}{\sigma}$$

Where:

- $\boldsymbol{\varepsilon}$: permittivity of the material
- $\boldsymbol{\sigma}$: conductivity

The **Debye length L_D** is the characteristic length over which charge imbalances are screened out in the material:

$$L_D = \sqrt{\frac{\varepsilon KT}{q^2 n}}$$

This parameter is crucial for understanding electric field screening in semiconductors and plays an important role in the design of junctions and interfaces.